Stat 412/512

Another Multivariate Response Study

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case1602: Oat Bran Study

Fiber might reduce serum cholesterol or

it might be that if you give someone a supplemental food (i.e. high fiber), they might just eat less of their usual food

Study designed to tease apart the “fiber” effect from the “different food” effect.

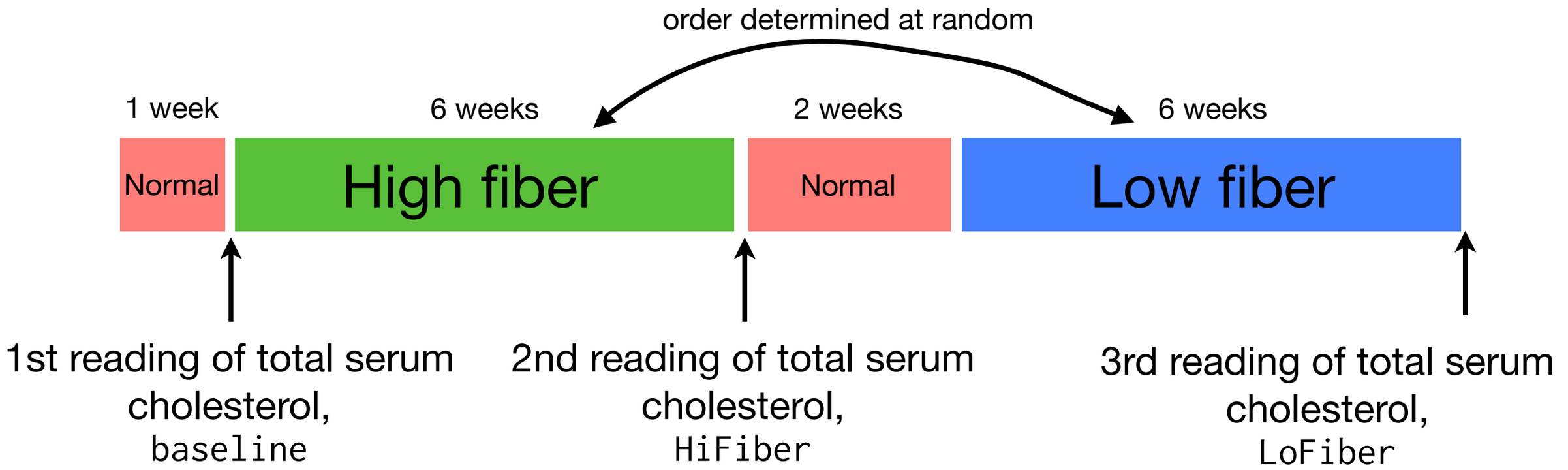
20 volunteers, two supplements: high fiber, low fiber.

One week of normal diet,

followed by six weeks of high fiber, two week normal diet then six weeks of low fiber (in a random order).

Response: Total serum cholesterol after each diet.

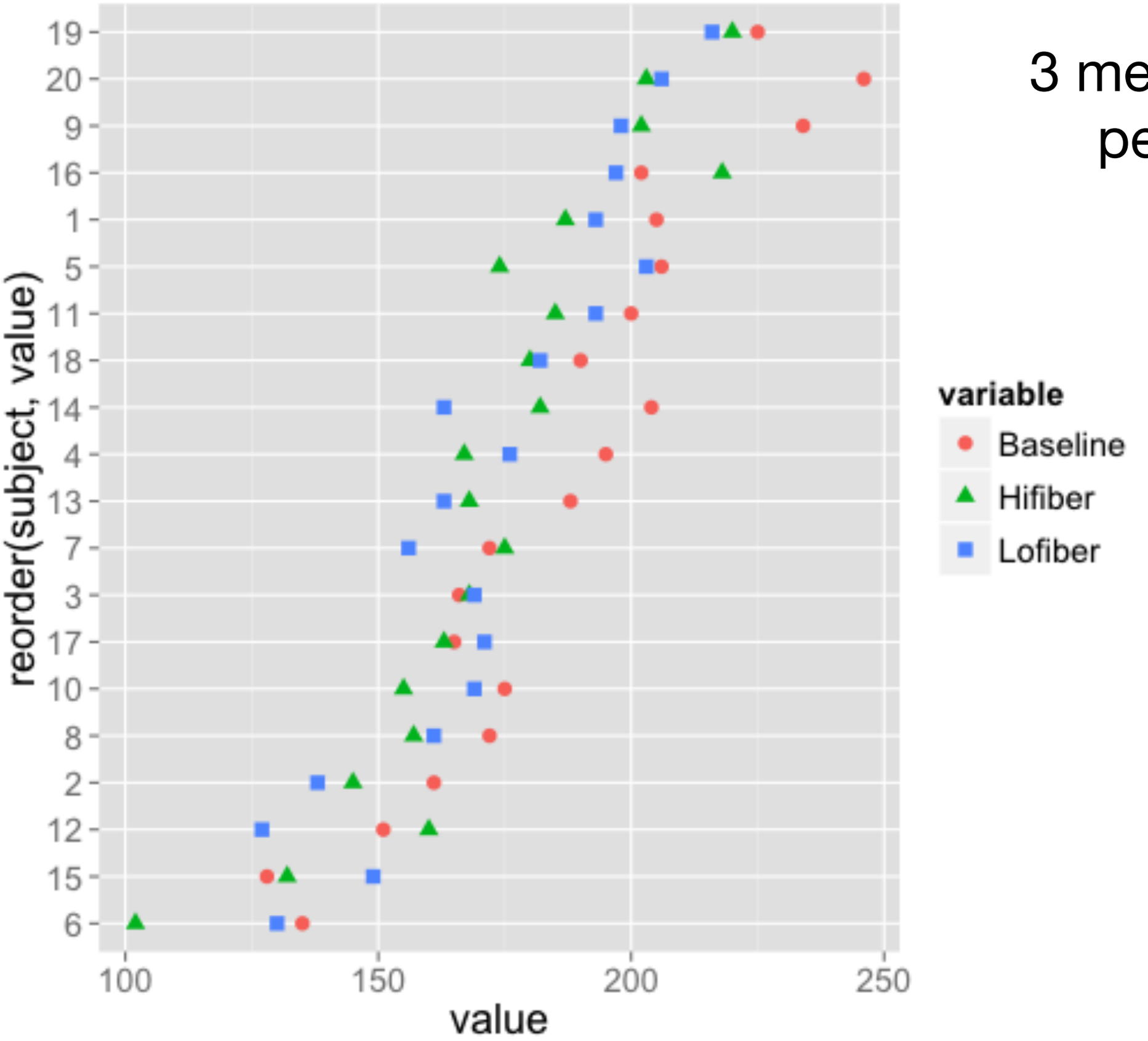
Crossover design
All subjects receive all treatments



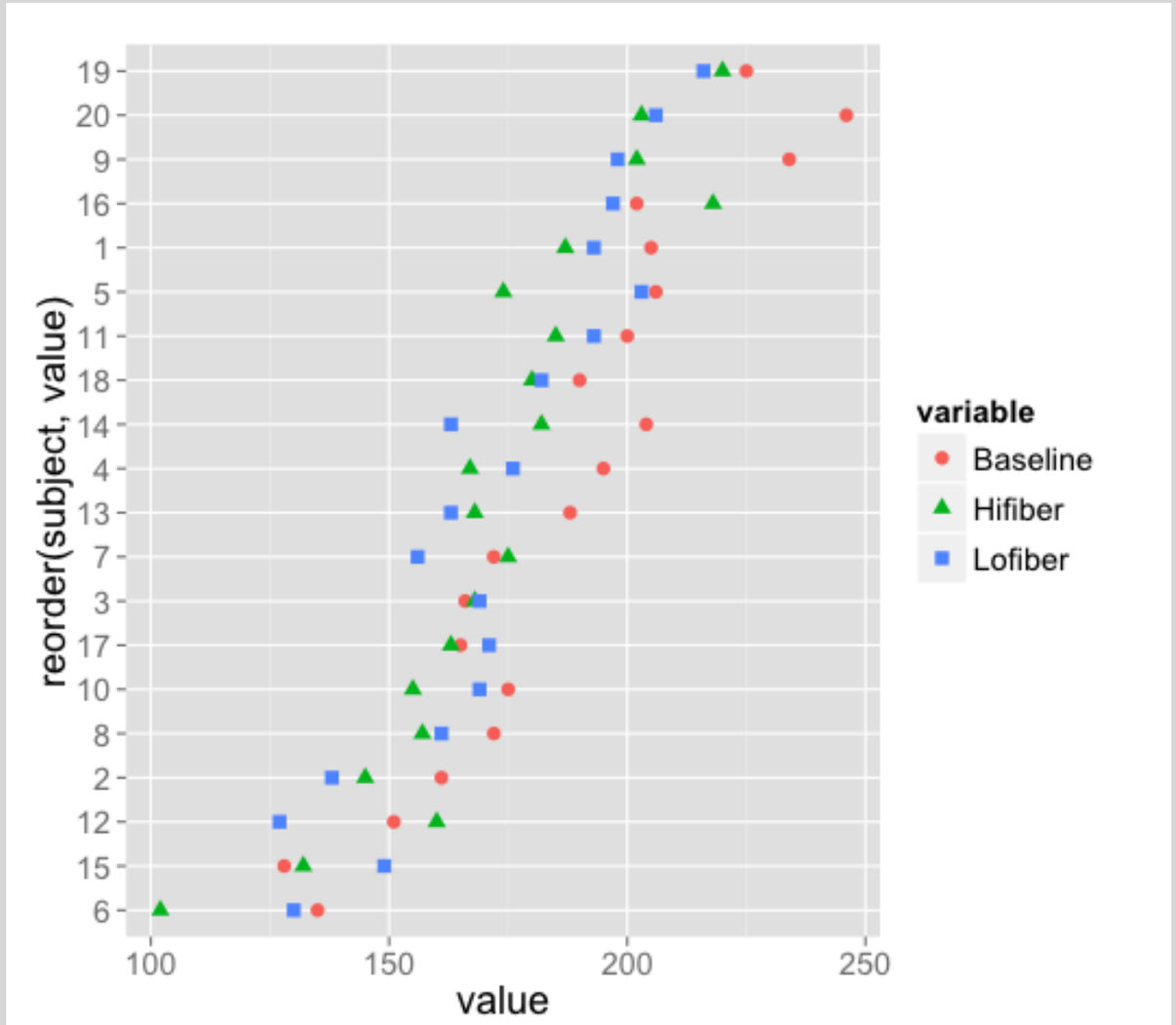
Does the high fiber supplement reduce cholesterol?

Does the high fiber supplement reduce cholesterol more than the low fiber supplement?

3 measurements per subject



Your turn



We have three measurements per person.

How could we summarise them into two numbers that directly address our questions?

Does the high fiber supplement reduce cholesterol?

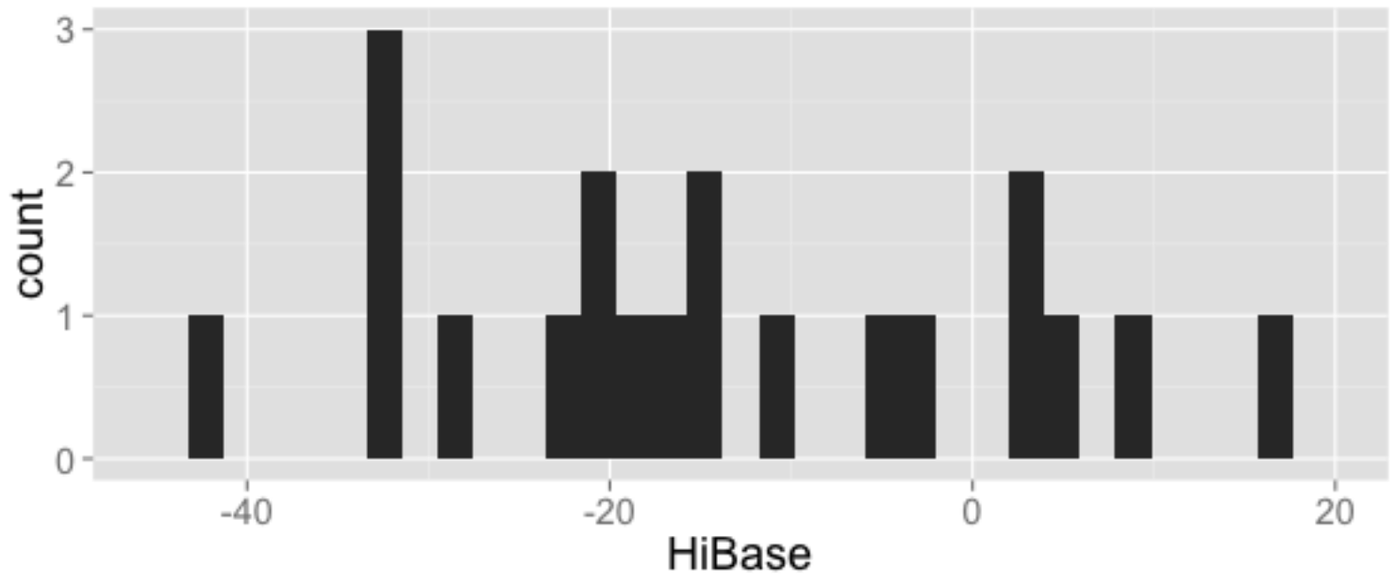
Does the high fiber supplement reduce cholesterol more than the low fiber supplement?

Cholesterol measurements for subjects after three diet regimes, and two comparison measures

Base Line	High Fiber	Low Fiber	Comparisons: Hi - Base	Comparisons: Hi - Lo	Fiber Order
205	187	193	-18	-6	HL
161	145	138	-16	7	HL
166	168	169	2	-1	HL
195	167	176	-28	-9	HL
206	174	203	-32	-29	HL
135	102	130	-33	-28	HL
172	175	156	3	19	HL
172	157	161	-15	-4	HL
234	202	198	-32	4	HL
175	155	169	-20	-14	HL
200	185	193	-15	-8	LH
151	160	127	9	33	LH
188	168	163	-20	5	LH
204	182	163	-22	19	LH
128	132	149	4	-17	LH
202	218	197	16	21	LH
165	163	171	-2	-8	LH
190	180	182	-10	-2	LH
225	220	216	-5	4	LH
246	203	206	-43	-3	LH
Average:			-13.85	-0.85	
Sample SD:			15.80	15.78	
Correlation:				+.516	

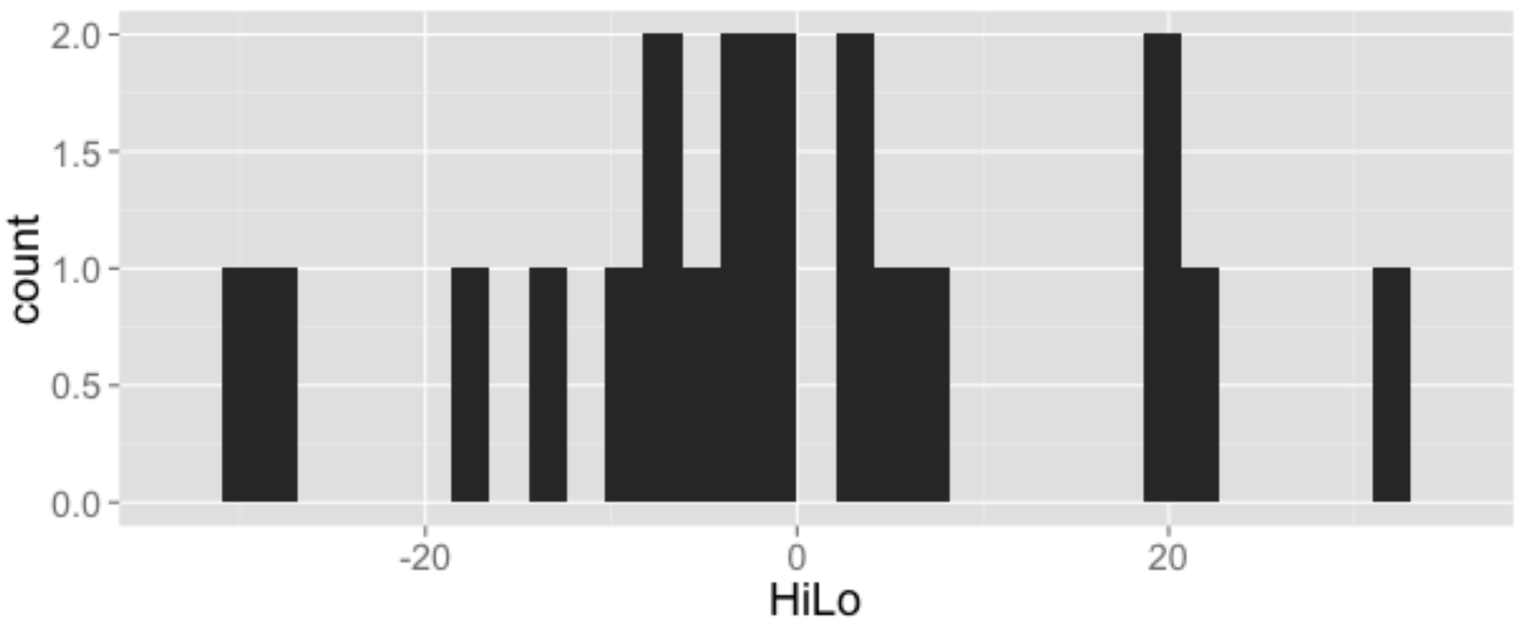
```
case1602$HiBase <- with(case1602, Hifiber - Baseline)
case1602$HiLo <- with(case1602, Hifiber - Lofiber)
```

Does the high fiber supplement reduce cholesterol?



$\delta_1 = 0$? paired t-test

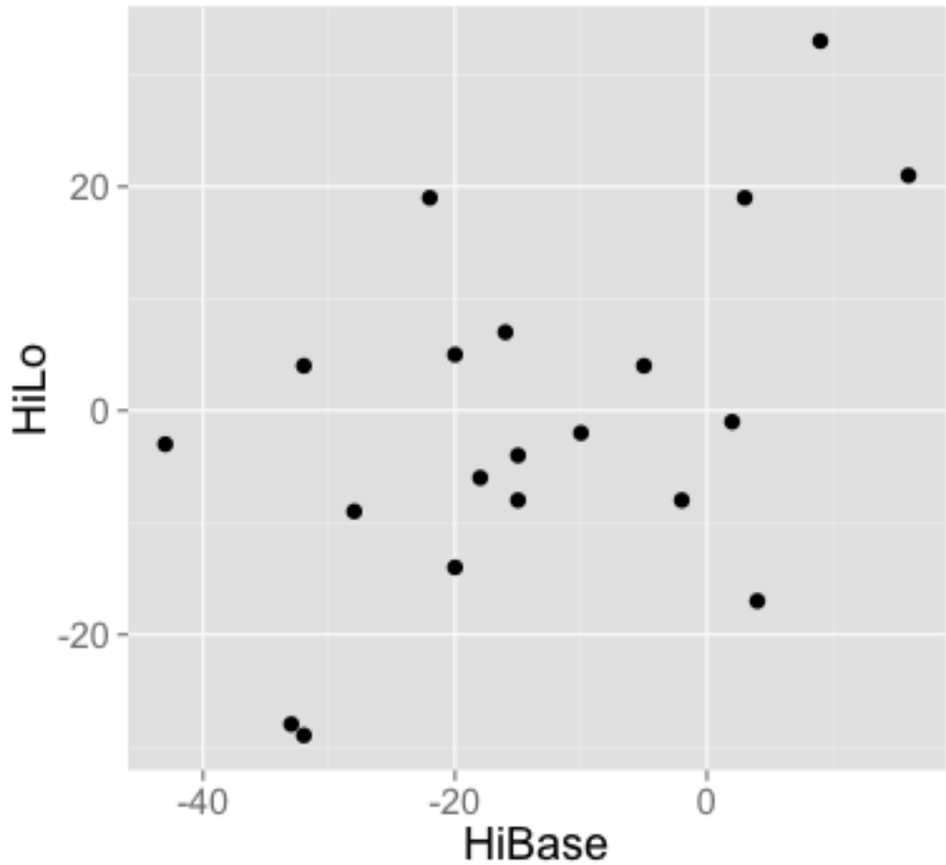
Does the high fiber supplement reduce cholesterol more than the low fiber supplement?



$\delta_2 = 0$? paired t-test

Two responses for two questions.

Again to account for correlation between the responses, we can use a Hotelling's T^2 adjustment



Ideal model for the Oat Bran study

The two responses are Normally distributed with means, δ_1 and δ_2, standard deviations, σ_1 and σ_2 and are correlated.

Subjects are randomly selected.

Observations of the two responses are independent across subjects.

Like the memory study but we only have one population, not two.

Hotelling's T^2

for a single bivariate sample

Null: $\delta_1 = 0$ and $\delta_2 = 0$

$$T^2 = \frac{t_1^2 + t_2^2 - 2rt_1t_2}{1 - r^2}$$

where r is the sample correlation between the two responses, and t_1 and t_2 are the individual t-statistics.

Under the ideal model,

$$F = \frac{n-2}{2(n-1)}T^2$$

has an F distribution with

2 and n - 2 degrees of freedom

Inferential tools applied to the oat bran data

(1) Summary statistics

	Hi-Base	Hi-Lo
Average:	-13.85	-0.85
Sample SD:	15.80	15.78
Sample Size:	20	
Correlation:	+.516	

(2) Calculate standard errors for the average effects

$$SE_1 = \frac{15.80}{\sqrt{20}} = 3.533 \quad ; \quad SE_2 = \frac{15.78}{\sqrt{20}} = 3.528$$

(3) Calculate individual one-sample t-statistics

$$t_1 = \frac{-13.85 - 0.0}{3.533} = -3.920 \quad ; \quad t_2 = \frac{-0.85 - 0.0}{3.528} = -0.241$$

(4) Combine into Hotelling's T^2

$$T^2 = \frac{(-3.920)^2 + (-0.241)^2 - 2(+.516)(-3.920)(-0.241)}{1 - (.516)^2} = 19.698$$

(5) Convert to an F-statistic

$$\text{F-statistic} = \frac{(18)}{(19)(2)}(19.698) = 9.331$$

(6) Look up the p-value

numerator df = 2 ; denominator df = n - 2 = 18

p-value = .0017 ← *provides convincing evidence that both mean effects are not zero*

(7) Construct 95% separate confidence intervals

$F_{2,18}(.95) = 3.555$ ← *from table*

$$T^2 = \frac{(19)(2)}{(18)}(3.555) = 7.505 \quad ; \quad \textit{Multiplier} = \sqrt{7.504} = 2.740$$

Hi - Base: $-13.85 \pm (2.740)(3.533) = -23.5$ mg/dl, -4.2 mg/dl

Hi - Lo: $-0.85 \pm (2.740)(3.529) = +8.8$ mg/dl, -10.5 mg/dl

Hotelling's CI multiplier, for a single bivariate sample

$$\sqrt{\frac{2(n-1)}{n-2} F_{2,n-2}(1-\alpha)}$$

It is estimated that the high-fiber supplement caused a 13.85 mg/dl reduction in cholesterol (95% CI 4.2 23.53 mg/dl).

The data are consistent with there being no difference in the high fiber and low fiber effects (95% CI -10.5 to 8.8 mg/dl).

Extensions

More than two responses:

Hotelling's T extends for the one and two sample case (but you probably want to use a computer to do the calculations).

More than one explanatory:

Both our examples only relied on one and two sample tools, there are extensions to multiple regression and ANOVA.

Next week…

I propose:

Mon - review topics you are interested in reviewing. Let me know what.

Wed - questions on last year's final

Fri - office hour instead, 76 Kidder